Euler Column Buckling Theory; Effects of Residual Stresses

MORGAN STATE UNIVERSITY
SCHOOL OF ARCHITECTURE AND PLANNING
LECTURE III
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WHAT IS RESIDUAL STRESS?

- Although Steel is considered to be homogenous material, the process of fabrication allows portions of an element to form differently than others.
  - Rolled shapes may go through the rollers hot or cold
  - For cold rolled, it is understood that the steel is exposed to stresses that bring it into its plastic region to have permanent deformation. When an element is stressed to the point that it deforms and it does not return to its original form, portion of the energy that was received remains within it. That is translated to a stress that is carried within the structure of that element. That is residual stress.
What is Residual Stress?

Continuing on causes of residual stress,
- For hot rolled elements, the residual stresses are not of the same scale, but they can still be significant, especially on larger elements. Those are developed by the uneven rate of cooling in different areas. Areas that cool quicker, such as the middle of the web or the tips of flanges of a W-Section tend to have residual compressive stress, whilst areas that cool slower, such as the intersections of web and flange, develop residual tensile stress.

Quick cooling

Slow cooling

What is Residual Stress?

- Residual stress is addressed as drops of the scale of 10-15ksi may have an effect on the reliability of a design. The greatest reductions in strength are noticed in columns that have a "Slenderness ratio" between 70 – 90.
- Slenderness ratio is the result of the division of the effective length "L" over the radius of gyration "r"
  - The former is essentially the length of the element multiplied by the "k" factor
  - The latter is a factor that can be found in the AISC User's Manual.
Qualities of Different Shapes

- Some shapes are more practical to fabricate
- Some shapes have better response to compressive loads
- Some shapes handle bending better

Other advantages / disadvantages:
- Round columns have less surface to paint or fireproof
- Round columns have constant “r” and “I” values
- They have better torsional resistance and less resistance to wind loads
- Square or round columns are more economical and efficient unless moments play an important role, especially in larger structures
- Hollow columns are easier to keep clean, but also easier to be exposed to corrosion over W, S, or T shapes
**Buckling**

- Main difference of a compressive axially loaded member over a tensile axially loaded member is "buckling."
  - That is the "loss of compressive load carrying capacity resulting from a change in the geometric formation of a member"
  - A slight defect, or a slight eccentricity, may generate the deflection that will lead to a column's failure

NY City Transit released photos of Cortlandt St station the week of 24 September 2001. Subway columns are buckled from the impact, near the center of the station. All of this is gone. Source: [http://www.columbia.edu/~brennan/abandoned/Cort-damage-09.jpg](http://www.columbia.edu/~brennan/abandoned/Cort-damage-09.jpg)

**Buckling**

- Design equation:
  - The ultimate axial load is equal or less than the factored nominal strength
  \[ P_u \leq \Phi P_n \]

![Typical behavior of a steel column](http://www.columbia.edu/~brennan/abandoned/Cort-damage-09.jpg)
Strength of Isolated Columns

Euler's solution to theoretical elastic behavior:
- Based on the following assumptions:
  - The column is pin connected
  - It is perfectly straight
  - Load is perfectly axial
  - Behaves elastically and does not yield
  - No residual stresses
  - Bends and buckles about a principal axis w/out torsion.

\[ P_e \leq \Phi P_n \]

Strength of Isolated Columns

Euler's elastic buckling:
- The buckled shape resembles \( \frac{1}{2} \) a sinusoidal distribution.
- The buckling load \( P_e \) is proportional to the Moment of Inertia of the element
- The buckling load is inversely proportional to the square value of the length of the element \( (L^2) \)
  - The longer the element the more susceptible to buckling
- Buckling is proportional to the Young's modulus of elasticity but independent of the yield strength of the material (\( F_y \))

\[ P_e = \frac{\pi^2 EI}{L^2} \]
Consider the Effects of Axial Load on a W Shape

- X & Y are Principal Axes
- \( I_{\text{max}} = I_x \) (Strong Axis)
  - Higher Moment of Inertia
- \( I_{\text{min}} = I_y \) (Weak Axis)
  - Lower Moment of Inertia
- Buckling will be about the (y) weak axis, unless the weak axis is restrained.

\[ \pi^2 \frac{EI_x}{L^2} \]
\[ \pi^2 \frac{EI_y}{L^2} \]
Addressing Euler's Buckling Load w.r.t. Stress

- Stress can be viewed as Load (P) divided by Area (A)
  - If we divide both sides of Euler's equation by Area →
    \[ \frac{P}{A} = \frac{\pi^2 EI}{AL^2} \]
  - But since the radius of gyration \((r)\) is equal to the square root of moment of inertia \((I)\) divided by area \((A)\)... \[ r = \sqrt{\frac{I}{A}} \quad \text{OR} \quad r^2 = \frac{I}{A} \]
  - Thus Euler's elastic buckling stress is:
    \[ F_e = \frac{\pi^2 E r^2}{L^2} \quad \text{OR} \quad F_e = \frac{\pi^2 E}{\left(\frac{L}{r}\right)^2} \]

Bringing Back the Factor of Boundary Conditions

<table>
<thead>
<tr>
<th>Buckled shape of column is shown by dashed line</th>
<th>Theoretical K value</th>
<th>Recommended design value when ideal conditions are approximated</th>
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<tbody>
<tr>
<td>(a)</td>
<td>0.65</td>
<td>0.80</td>
</tr>
<tr>
<td>(b)</td>
<td>1.0</td>
<td>1.2</td>
</tr>
<tr>
<td>(c)</td>
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<td>(d)</td>
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<tr>
<td>(e)</td>
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<td>2.0</td>
</tr>
<tr>
<td>(f)</td>
<td>2.0</td>
<td>2.0</td>
</tr>
</tbody>
</table>

End condition code
- Rotation fixed
- Rotation free
- Translation fixed
- Translation free
**Load Deflection Behavior**

- The Length “L” divided by the radius of gyration “r” is the slenderness ratio of a column
- By plotting a graph of the stress in the ordinates and the slenderness ratio in the abscissa...
- r minimum corresponds to I minimum
- (L/r) max corresponds to r min
- Weaker axis of W section (lower I) controls in buckling

\[ \pi^2 \frac{E}{I} \frac{L^2}{r} \]

**Load – Deflection Behavior**

- Effects of deflection
  - The column bends as soon as it is loaded, i.e. buckling is not an instantaneous effect.
  - There is already stress in the column before loading.
  - Based on Elastic theory (material does not yield) P is asymptotic to PE. No loss of strength due to deflection
  - In reality material yields, and the additional bending stress from deflection causes earlier yielding and loss of strength
  - Small deflection → little loss of strength, and vice versa

\[ P_E = \pi^2 \frac{E I}{L^2} \]
Effects of Load Eccentricity

- Effects of eccentricity (eccentrically applied loading) are identical to the effects of $\Delta 0$

Definition and Effects of Residual Stresses

- Residual stress definition
  - They are developed within a member during manufacturing.
  - They are self equilibrating (their sum is zero) as they exist in the absence of any external loading.
  - They are generated by:
    - Uneven cooling of hot rolled elements
    - Uneven cooling of welded built up elements
    - Cold forming or cambering of members
    - Punching, shearing, or cutting
    - Welding at specific points

- Effect
  - Varying behavior at specific points, areas, or along an axis
**CAUSES FOR RESIDUAL STRESS**

- **Cooling of Rolled Shapes**
  - The uneven rate of cooling of the cross section.
  - Member is allowed to cool slowly. Some portions (e.g. flange tips) cool quicker because they have more surface exposed to air
  - Typically residual stresses are:
    - Quick cool → Compressive
    - Slow cool → Tensile
  - Residual stresses are normal, not shear stresses
  - Residual stresses are higher on welded shapes than rolled shapes

\[ \int \sigma_{res} \, dA = 0 \]
\[ \sigma_{res_{max}} \approx 10^{ksi} \text{ to } 15^{ksi} \]

**THE STUMP COLUMN TEST**

- Residual stresses reduce the stiffness of a member
  - Investigated by testing a “stub column,” i.e. too short to buckle.
  - If there are no residual stresses, all fibers of the cross section yield simultaneously when the applied load reaches

\[ \frac{P}{A} = F_y \]
The Stump Column Test

- Residual stresses reduce the stiffness of a member
  - If residual stresses are present the first parts of the element that will yield are the tips of the flanges.
  - Then the effect will extend further beyond the tips of the flanges and the central portion of the web.
  - And eventually the whole section will yield.
  - Although the maximum load will still be \( P = A \times F_y \), the load deflection curve is not the same.

Using the Tangent Modulus

- The \( P \) vs \( \Delta \) curve..
  - Can be replotted in the form of average applied stress vs strain.
  - \( E_t \) is a measure of the cross section's average stiffness, considering that portions of the cross section are yielded, while others are still elastic.

\[
\sigma_{av} = \frac{P}{A} = \text{average applied stress}
\]
\[
\varepsilon = \frac{A}{L_0} = \text{applied strain}
\]
Effect of Residual Stresses on Column Strength

Consider a column that is initially perfectly straight

- The buckling load can be obtained using the “tangent modulus theory” that was just discussed.
- The buckling load can be computed using Euler’s equation, but replacing E with Et.
- The resulting buckling load is referred to as “Tangent Modulus buckling load”
- Similarly we can define the “Tangent Modulus buckling stress”
- This leads to two classes of buckling:
  - Elastic, and
  - Inelastic

\[ P_t = \frac{\pi^2 E_t L}{(K \times L)^2} \]
\[ F_t = \frac{P_t}{A} = \frac{\pi^2 E_t}{(K \times L)^2} \]

Elastic / Inelastic Buckling

- Elastic
  - No yielding of the cross section occurs prior to buckling and Et=E at buckling
  - \( F_E = \frac{\pi^2 E}{(K \times L)^2} \) predicts buckling

- Inelastic
  - Yielding occurs on portions of the cross section prior to buckling and there is loss of stiffness.
  - \( F_r = \frac{\pi^2 E_t}{(K \times L)^2} \) predicts buckling
Strength of Columns

- The discussion that was held until now indicates that the strength of a column is dependent upon the following:
  - Slenderness \( \lambda \): \( \frac{L}{r} \)
  - End restraint: \( K \) factor
  - Eccentricity (loading or form)
  - Yielding and Residual stresses

- All of the above factors need to be addressed in order to determine the strength of a real column but there are two approaches to do that:
  - Experiments (we shall not engage in this!)
  - Numerical Analysis

Numerical Method of Analysis

- The AISC provides a series of equations that allow us to compute the column strength:
  - Nominal compressive strength \( P_n = A_g \times F_{cr} \)
    where \( A_g \) is Area gross, and \( F_{cr} \) is the critical or buckling stress
  - Design compressive strength \( \Phi P_n = \Phi \times A_g \times F_{cr} \)
    where \( \Phi \) is the factor of safety and it is equal to 0.9
  - Criterion for design \( P_n \leq \Phi \times P_n \)
**Numerical Method of Analysis**

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  - Nominal compressive strength: \( P_n = A_g \times F_{cr} \)
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    where \( \Phi \) is the factor of safety and it is equal to 0.9
  - Criterion for design: \( P_u \leq \Phi \times P_n \)

**Computing the Nominal Compressive Strength**

- The definition is: \( F_{cr} = \frac{\pi^2 E}{(KL)^2} \) for Euler's buckling Stress

- When \( \frac{KL}{r} \leq 4.71 \sqrt{\frac{E}{F_y}} \) OR \( \frac{F_y}{F_e} \leq 2.25 \) → Inelastic Buckling
  \[ F_{cr} = 0.658 \left( \frac{F_y}{F_e} \right) F_y \]  
  (E3-2)

- When \( \frac{KL}{r} > 4.71 \sqrt{\frac{E}{F_y}} \) OR \( \frac{F_y}{F_e} > 2.25 \) → Elastic Buckling
  \[ F_{cr} = 0.877 F_e \]  
  (E3-3)

For elastic buckling we adopt 0.877 times the Euler's formula, accounting for geometric imperfections.

- Note that \( F_{cr} \) is independent of \( F_y \)
**THE RED LINE FOR ELASTIC / INELASTIC**

- **Inelastic Buckling**
  \[ F_{cr} = 0.658 \left( \frac{F_y}{F_E} \right) \]
  \[ F_{cr} = 0.658 \left( \frac{F_y}{\pi^2 E (KL/r)^2} \right) \]

  - Note: As KL/r → 0 Fcr → Fy

- **Elastic Buckling**
  \[ F_{cr} = 0.877 F_E = 0.877 \pi^2 E \left( \frac{KL}{r} \right)^2 \]

**THE RED LINE FOR ELASTIC / INELASTIC**

- Taking the limit: \( \frac{KL}{r} = 4.71 \sqrt{\frac{E}{F_y}} \) for 36 and 50 grade steel

  - \( 4.71 \sqrt{\frac{29,000}{36}} = 133.681 \) → 36 ksi gives 133.7
  - \( 4.71 \sqrt{\frac{29,000}{50}} = 113.432 \) → 50 ksi gives 113

- Take a typical column e.g. W12x53, of Lu=12' and r=2.48". With K of 1.0 this W section will give KL/r=58. In either of the grades of steel this column will buckle in the inelastic range.
 Relation of Critical Stress and Slenderness Ratio

At $\frac{KL}{r} = 4.71 \sqrt{\frac{E}{F_y}}$

$F_E = \frac{\pi^2 E}{(KL/r)^2} = \frac{\pi^2 EF_y}{4.71^2 E} = 0.44F_y$

Thus $\frac{F}{F_E} = 2.25$

$F_{cr} = 0.877F_E = 0.877 \times 0.44F_y = 0.39F_y$

The transition from elastic to inelastic occurs at an applied axial compression stress of 0.39$F_y$.

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Basic Procedures for Analysis

Given the shape, the K factor, the Length and the type of steel can we determine the $\Phi_{pn}$?

- Well, ...... $\Phi_{pn} = \Phi \times A_g \times F_{cr}$
- $F_{cr}$ depends upon $KL/r$ and $F_y$,
- The $r$ we chose is the weaker one $\rightarrow (KL/r)$ max controls
  - Compute $(KL/r)x$ and $(KL/r)y$, and larger will govern
Wide Flange Shape subjected to axial loading

**Problem Statement:**
Determine the capacity in axial loading of the given W shape. The element is pinned at top and bottom with no intermediate bracing, therefore having an unbraced length of 15ft in both directions. Use A992 steel

<table>
<thead>
<tr>
<th>Area</th>
<th>Young's Modulus of Elasticity</th>
<th>Bolt diameter</th>
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</thead>
<tbody>
<tr>
<td>$A_g := 15.8\text{in}^2$</td>
<td>$E := 29000\text{ksi}$</td>
<td>$d_b := 0.875\text{in}$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Length:</th>
<th>Yield Stress:</th>
<th>Ultimate Strength:</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L_u := 15\text{ft}$</td>
<td>$F_y := 50\text{ksi}$</td>
<td>$F_u := 65\text{ksi}$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>radius of gyration $y$</th>
<th>radius of gyration $x$</th>
<th>K factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r_y := 2.56\text{in}$</td>
<td>$r_x := 4.37\text{in}$</td>
<td>$K := 1$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Factor of Safety phi</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi := 0.9$</td>
</tr>
</tbody>
</table>

**Solution Method 1:** Using Chapter E Equations:

1) Determining the governing slenderness ratio

$$\lambda_x := \frac{KL_u}{r_x} \left( \frac{15\text{ft} \cdot 12\text{in}}{4.37\text{ft}} \right)$$

$$\lambda_x = 41.19$$

$$\lambda_y := \frac{KL_u}{r_y} \left( \frac{15\text{ft} \cdot 12\text{in}}{2.56\text{in}} \right)$$

$$\lambda_y = 70.313$$

$r := \min(r_x, r_y)$  
$governing\ radius\ of\ gyration = 2.56\text{in}$

The above was already obvious but it was carried on just to "academically" justify the numbers

2) Calculating Euler's Buckling Stress

$$F_E := \frac{2 \cdot E}{r^2} \left( \frac{15\text{ft} \cdot 12\text{in}}{2.56\text{in}} \right)$$

$$F_E := 57.894\text{ksi}$$

3) Determining if the buckling will be elastic or inelastic.

Buckling := \[ \left[ \frac{KL_u}{r} \right] \leq 4.71 \sqrt{\frac{E}{F_y}} \]

When \[ \frac{KL}{r} \leq 4.71 \sqrt{\frac{E}{F_y}} \]  
OR \[ \frac{F}{F_y} \leq 2.25 \] \rightarrow Inelastic Buckling

Alternatively we can also follow the process below:

$$\frac{F_y}{F_E} = 0.864$$

Buckling := \[ \left[ \frac{F_y}{F_E} \right] \leq 2.25, "Inelastic", "Elastic" \]

Buckling = "Inelastic"

4) Calculating the Buckling Stress ($F_{cr}$) and the load capacity of the section:

$$F_{cr} := \left[ 0.658 \left( \frac{F_y}{F_E} \right) \right] F_y \left[ 0.658 \left( \frac{50\text{ksi}}{57.89\text{ksi}} \right) \right] 50\text{ksi}$$

$$F_{cr} = 34.832\text{ksi}$$

$$\Phi P_n := \Phi \cdot A_g \cdot F_{cr}$$

$$0.9 \cdot 15.8\text{in}^2 \cdot 34.832\text{ksi}$$

$$\Phi P_n = 495.314\text{kip}$$
Solution Method 2: Using Table 4-22:

1) Determining the governing slenderness ratio

\[ \lambda_y := \frac{K-L_u}{r_y} \left( \frac{16 \text{ft} \cdot 12 \text{in}}{\text{ft}} \right) \]

\[ \lambda_y = 70.313 \]

2) Using table we locate the KL/r value corresponding to the Fy used for factored critical stress:

The value indicated would be between 31.1 and 31.4. Let's take 31.3

\[ \Phi F_{cr} := 31.3 \text{ksi} \]

Note: From our previous calculations:

\[ F_{cr} = 34.832 \text{ksi} \]

Therefore:

\[ \Phi F_{cr} = 31.349 \text{ksi} \]

3) Calculating the capacity of the element:

\[ \Phi P_n := \Phi A_g F_{cr} \]

\[ 0.9 \cdot 15.8 \text{in}^2 \cdot 34.832 \text{ksi} \]

\[ \Phi P_n = 495.314 \text{kip} \]
Solution Method 3:
Using Table 4-1 for W shapes pp 4-12 to 4-23:
Oh you will love this one! All you need is the unbraced length and the shape:

<table>
<thead>
<tr>
<th>Shape</th>
<th>54</th>
<th>49</th>
<th>45</th>
<th>39</th>
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</thead>
<tbody>
<tr>
<td>Design</td>
<td>( P_{u}/\Omega_{c} )</td>
<td>( \phi_{c}P_{u} )</td>
<td>( P_{u}/\Omega_{c} )</td>
<td>( \phi_{c}P_{u} )</td>
<td>( P_{u}/\Omega_{c} )</td>
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<td>91.1</td>
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</tr>
</tbody>
</table>

Properties:

- \( P_{w0} \), kips: 69.1, 104, 60.1, 90.1, 65.3, 98.0, 104, 81.1, 45.2, 67.8
- \( P_{wa} \), kips/ln: 12.3, 18.5, 11.3, 17.0, 11.7, 17.5, 10.5, 15.8, 9.67, 14.5
- \( P_{wb} \), kips: 112, 168, 86.6, 130, 94.2, 142, 68.7, 103, 53.7, 80.7
- \( P_{ue} \), kips: 70.8, 106, 58.7, 86.2, 71.9, 108, 52.6, 79.0, 35.4, 53.2
- \( L_{w} \), ft: 9.04, 8.97, 7.10, 6.99, 6.85
- \( L_{w} \), ft: 33.6, 31.6, 26.9, 24.2, 21.8
- \( A_{w} \), in²: 15.8, 14.4, 13.3, 11.5, 9.71
- \( I_{w} \), in⁴: 303, 272, 248, 209, 171
- \( L_{w} \), in: 103, 93.4, 53.4, 45.0, 36.6
- \( A_{w} \), in²: 2.56, 2.54, 2.01, 1.98, 1.94
- \( I_{w} \), in⁴: 1.71, 1.71, 2.15, 2.16, 2.16
- \( P_{ue}(kL)^{2}/10^{4}, \text{k-in}^{2} \): 8670, 7790, 7100, 5980, 4890
- \( P_{ue}(kL)^{2}/10^{4}, \text{k-in}^{2} \): 2950, 2670, 2150, 1290, 1050

ASD | LRFD
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\( \Omega_{c} = 1.67 \) | \( \phi_{c} = 0.90 \)

Note: Heavy line indicates \( KL/\gamma_{f} \) equal to or greater than 200.